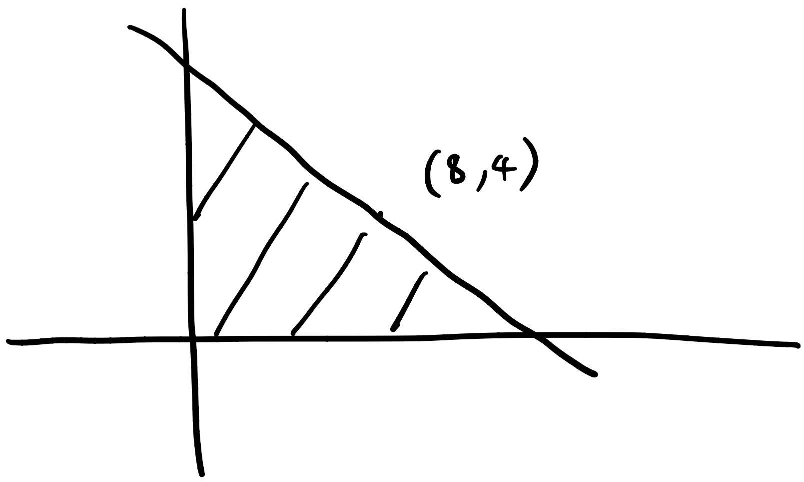
Consider triangles formed by lines passing through the point $(8,4)$, the $x$-axis, and the $y$-axis.

Find the dimensions that minimize area.



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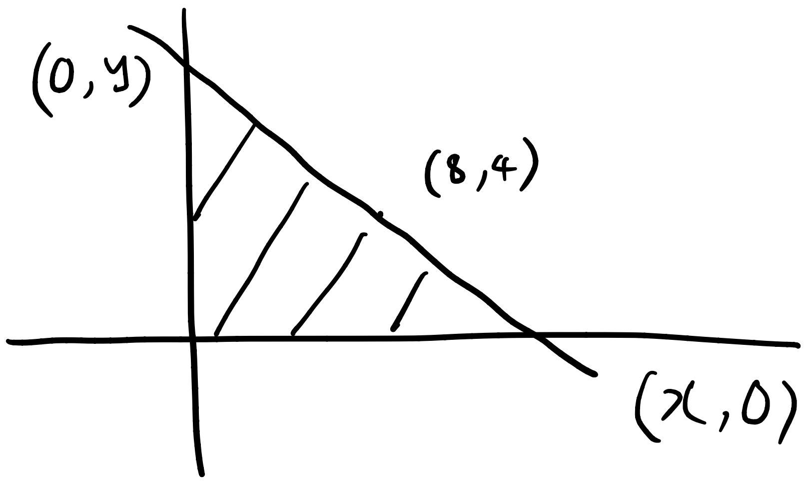



$$0 - y = m(x - 0)$$

$$-y = \frac{0-4}{x-8}(x)$$

$$y = \frac{4x}{x-8}$$

$$A = \frac{1}{2}xy$$

$$= \frac{1}{2}\,\frac{4x^2}{x-8}$$

$$= \frac{2x^2}{x-8}$$

$$= 2x + 16 + \frac{128}{x-8}$$

$$A' = \frac{4x(x-8) - 2x^2(1)}{(x-8)^2}$$

$$= \frac{2x^2 - 32x}{(x-8)^2}$$

$$A' = 0 \Rightarrow 2x^2 - 32x = 0$$

$$x(x-16) = 0$$

$$x = 0, 16$$

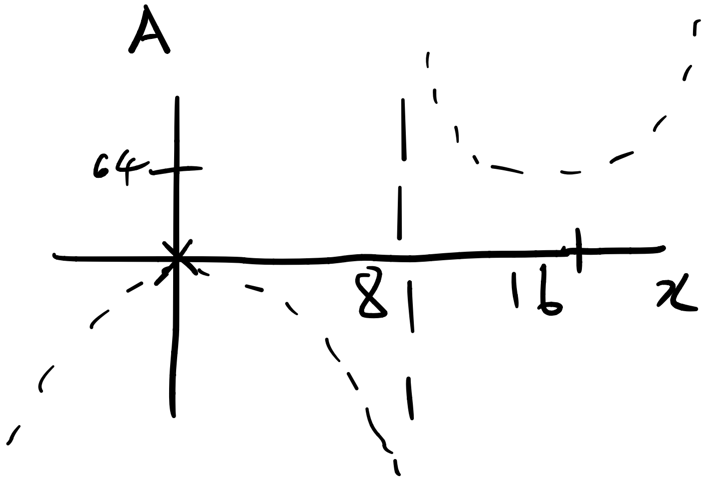
$\because$ There is 0 area at $x = 0$

$\therefore$ $x = 16$

$$\lim_{x \to 8^-} A = \frac{2(8^-)^2}{8^- - 8}$$

$$= \frac{128}{-\Delta} = -\infty$$

$$\lim_{x \to 8^+} A = \infty$$




$$A = 2(16) + 16 + \frac{128}{16-8}$$

$$= 32 + 16 + \frac{128}{8}$$

$$= 48 + 16$$

$$= 64$$

$$y = \frac{4(16)}{16-8} = \frac{64}{8} = 8$$

$$m = -\frac{4}{(16)-8} = -\frac{1}{2}$$

$$\lim_{x \to \infty} A = \infty$$

$$\lim_{x \to -\infty} A = -\infty$$

$\therefore$ At $x = 16$ and $y = 8$, the area of triangle is minimized.